Normal Distributions
Histograms and Area

**FIGURE 2-15**

Types of Histograms

(a) Typical symmetrical histogram

(b) Typical uniform or rectangular histogram

(c) Typical skewed histogram

(d) Typical bimodal histogram

Skewed left

Skewed right
FIGURE 6-1
A Normal Curve

Highest point of curve
Downward cup

Transition points

Upward cup

$\mu - 3\sigma$  $\mu - 2\sigma$  $\mu - \sigma$  $\mu$  $\mu + \sigma$  $\mu + 2\sigma$  $\mu + 3\sigma$
Curves that are not normal

Each of the curves in Figure 6-2 fails to be a normal curve. Give reasons why these curves are not normal curves.

**FIGURE 6-2**
Important properties of a normal curve

1. The curve is bell-shaped with the highest point over the mean $\mu$.
2. It is symmetrical about a vertical line through $\mu$.
3. The curve approaches the horizontal axis but never touches or crosses it.
4. The transition points between cupping upward and downward occur above $\mu + \sigma$ and $\mu - \sigma$. 
Empirical rule

For a distribution that is symmetrical and bell-shaped (in particular, for a normal distribution):

Approximately 68% of the data values will lie within one standard deviation on each side of the mean.

Approximately 95% of the data values will lie within two standard deviations on each side of the mean.

Approximately 99.7% (or almost all) of the data values will be within three standard deviations on each side of the mean.
FIGURE 6-5

Area Under a Normal Curve

- 2.35% between $\mu - 3\sigma$ and $\mu - 2\sigma$
- 13.5% between $\mu - 2\sigma$ and $\mu - \sigma$
- 34% between $\mu - \sigma$ and $\mu$
- 34% between $\mu$ and $\mu + \sigma$
- 13.5% between $\mu + \sigma$ and $\mu + 2\sigma$
- 2.35% between $\mu + 2\sigma$ and $\mu + 3\sigma$

- 68% below $\mu - \sigma$ and above $\mu + \sigma$
- 95% below $\mu - 2\sigma$ and above $\mu + 2\sigma$
- 99.7% below $\mu - 3\sigma$ and above $\mu + 3\sigma$
Empirical rule

The yearly wheat yield per acre on a particular farm is normally distributed with mean \( \mu = 35 \) bushels and standard deviation \( \sigma = 8 \) bushels.

(a) Shade the area under the curve in Figure 6-7 that represents the probability that an acre will yield between 19 and 35 bushels.

(b) Is the area the same as the area between \( \mu - 2\sigma \) and \( \mu \)?

FIGURE 6-7

(c) Use Figure 6-5 to find the percentage of area over the interval between 19 and 35.

(d) What is the probability that the yield will be between 19 and 35 bushels per acre?
(Number of standard deviations between the measurement and the mean) = \left( \frac{\text{Difference between the measurement and the mean}}{\text{Standard deviation}} \right)

Written in symbols, this formula is

\[ z = \frac{x - \mu}{\sigma} \]

**Definition** The \( z \) value or \( z \) score tells us the number of standard deviations the original measurement is from the mean. The \( z \) value is in *standard units*. 
Standard score

A student has computed that it takes an average (mean) of 17 minutes with a standard deviation of 3 minutes to drive from home, park the car, and walk to an early morning class.

(a) One day it took the student 21 minutes to get to class. How many standard deviations from the average is that? Is the $z$ value positive or negative. Explain why it should be either positive or negative.
(b) Another day it took only 12 minutes for the students to get to class. What is this measurement in standard units? Is the z value positive or negative? Why should it be positive or negative?

(c) Another day it took 17 minutes for the students to get to class. What is this measurement in standard units? Is the z value positive or negative? Why should it be positive or negative?

What percent of students can make it to class in 12 minutes?
Don’t forget the negative

Using your Z-Score

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