As the Crow flies: Applying the Pythagorean Thm to latitude and longitude

ED5661 Mathematics and Navigation Teacher Institute
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Target: Algebra and Geometry, grades 8-10

Time Required: 1 – 2 Class periods (50 – 100 minutes)

Lesson Overview: Students will determine distances (in nautical miles) between two points listed in latitude and longitude in both cardinal directions (N/S, E/W) and straight line (“as the crow flies”) distances. The overall purpose of this lesson is to provide students with an opportunity to apply the Pythagorean thm in a real world situation as well as introduce students to various types of units in use around the world.

Sources:
Teaching Mathematics through Navigation Teacher Institute, Michigan Tech University, Summer 2010
Boat Navigation for the Rest of Us, Captain Bill Brogdon
Various websites listed throughout the document.

Learning Objectives: Students will apply understandings of latitude and longitude, as well as the Pythagorean thm and unit conversion to determine the distance between multiple points via various paths.

After completing this lesson, students will be able to:

1) determine the distances between two points of lat and long
   1. N/S, E/W and Direct
   2) Convert minutes of latitude to nautical miles
   3) Convert minutes nautical miles to statute miles
   4) Apply the Pythagorean thm to real world problems

Materials Needed:
Students will work in groups of 3-4

The instructor will need a projector and laptop loaded with:

Rose Point Navigation Systems: Coastal Explorer Viewer and Express Editions (Free / 99.2 mb download). This software is available from http://rosepointnav.com/CoastalExplorerExpress/default.htm

The appropriate NOAA RNC (Raster Navigational Chart) for the indicated areas. For this lesson, that includes chart 14862 and chart 14863. The lake Huron chart 14860 may also be useful as an overall reference for the instructor. NOAA RNC’s for the Great Lakes (with online viewer) are available here: http://www.charts.noaa.gov/OnLineViewer/GreatLakesViewerTable.shtml
Note they can also be downloaded from this site for offline use.

Alternately, printouts or printed versions of the charts would be sufficient if enough can be obtained for the class. These can be purchased online, or out of date charts may be acquired from charter captains in the area.

Calculators or Spreadsheet software, as well as pencils, paper, etc...

**Knowledge for Success in this Lesson**

**Vocab Needed:**

- Nautical Mile - a unit of length corresponding approximately to one minute of arc of latitude along any meridian. By international agreement it is exactly 1,852 metres (approximately 6,076 feet).
- Bearing – The direction to an object as measured from your current position
- Course – The direction in which you intend to head
- Direction (true vs magnetic) – Angle, measured clockwise from one point to another. Directions are true when measured from the north pole, and magnetic M when measured from the magnetic north pole
- Heading – The direction that you are currently pointing
- Range – Distance to an object

**Formulas and Conversions:**

Pythagorean Thm: $a^2 + b^2 = c^2$
1 nm = 6076 ft
1 statute mile = 5280 ft
1 nm = 1 minute of latitude
1 minute of latitude = 60 seconds (lat)
1 deg latitude = 60 minutes (lat)

**Focus Question:** We've all used latitude and longitude through various courses, and living on the great lakes most of us have been boating at one point or another – how do we determine the actual distance between two different points of lat and long? Additional item for discussion – you're at sea. Your GPS dies. What do you do?

**Michigan Mathematics Standards Addressed:**

**General:**

Locate and describe objects in terms of their positions
Locate and describe objects in terms of orientation and relative position including displacement
Use concepts of position, direction and orientation to describe the physical world and solve problems
Analyze problems that can be modeled by functions and determine strategies for solving problems

**Specific Strands:**

L2.1.2 Calculate fluently with numerical expressions involving exponents; use the rules of exponents; evaluate numerical expressions involving rational and negative exponents; transition easily between roots and exponents

A1.2.2 Associate a given equation with a function whose zeros are the solutions of the equation.

A1.2.6 Solve power equations and equations including radical expressions; justify steps in the solution and explain how extraneous solutions may arise.

A3.2.5 Relate exponential functions to real phenomena

G1.2.3 Know a proof of the Pythagorean thm and its converse to solve multi-step problems.

G.GS.08.01 Understand at least one proof of the Pythagorean theorem and it's converse to solve applied problems including perimeter area and volume

G.L0.08.02 Find the distance between two points on the coordinate plane using the distance formula; recognize that the distance formula is an application of the Pythagorean thm.

**Classroom / Field Activities:**

Background / Points For Classroom Discussion:

Discuss the Storm of 1913 and local shipwrecks
http://www.pointeauxbarqueslighthouse.org/preserve/diving.cfm
http://www.michiganpreserves.org/thumb.htm
Discuss the Charity Island Lighthouse – Website: 

Discuss the Pointe Aux Barques Lighthouse – Website: 
http://www.terrypepper.com/lights/huron/ptauxbarques/index.htm

Emphasize the importance of knowing your position, and the value of having multiple inputs (Radar, GPS, ChartPlotter, Charts).

Work an illustration problem on the board for the students to follow:

EX:

The Pte Aux Barques Lt is located at 44 deg 1.235 N and 82 degrees 49.361 minutes West. The bow of the Daniel J. Morell lies in 205 feet of water at GPS coordinates 44 deg 18.320 minutes N, 082 degrees 45.161 minutes West. How far does the bow lie from the lighthouse? If you were to go on a dive to the bow, and your boat cruises at 18 knots, how long would it take you to get onsite?

Solve for students.

Begin Group Work

1) Form Groups
2) Ensure that each group has a calculator, pencil, paper, chart (if needed) or laptop
3) Pass out the question / problem sheet
4) Use Coastal Explorer Viewer to throw up a chart of the area.
5) Allow group work time, while roaming to answer questions and keep students on task.

Assessment:
Collect handouts, display on the projector / Elmo and have the groups explain their work to the class including their reasons for making their individual/specific choices where applicable.

Dissemination:
Lesson plan is available at – http://www.trusock.com/ED5661 in PDF and DOC format.
1) The abandoned Charity Island lighthouse is located at 44 deg 02.275 minutes North, 83 deg 26.465 minutes West. Charts show that there is a wreck lying in 14 feet of water at 44 degrees 01.799 minutes North, and 83 degrees 28.271 minutes west. How far north or south from the lighthouse does the wreck lie? How far east or west? What is the distance directly from the wreck to the lighthouse?

2) There is another wreck lying about 2.5 nautical miles off Sand Point in about 4 feet of water. The coordinates of this one are: 43 degrees 55.495 minutes North, 83 degrees 27.637 minutes West. How far north or south from the lighthouse does the wreck lie? How far east or west? What is the distance directly from the wreck to the lighthouse?

3) The bow of the Daniel J. Morell lies in 205 feet of water at GPS coordinates 44 deg 18.320 minutes N, 082 degrees 45.161 minutes West. The stern lies in 218 feet of water some distance away – 44 deg 15.478 minutes N, 82 deg 50.088 minutes W. How far north (or south) does the bow lie from the stern? How far east (or west) does the bow lie from the stern? What is the distance between the two?

4) Just off shore of Harbor Beach in 32 feet of water lies the Chickamauga. It's position in 45 deg 50.950 minutes N, 82 deg 37.430 minutes W. If you stand at the tip of the public dock, your coordinates are 43 deg 50.486 minutes N and 82 deg 38.637 minutes W. How far north (or south) does the wreck lie? How far east (or west)? What is the distance directly from you to the wreck?
The Wreck of the Daniel J Morell

580 ft long, 58 wide and 27 ft high

29 November 1966

Making the last run of the season with its sister ship the SS Edward Y. Townsend, the Morrell became caught in winds exceeding 70 mph (110 km/h) and swells that topped the height of the ship (20-25 foot waves). During the early morning hours, the Townsend made the decision to take shelter in the St. Mary's River, leaving the Morrell alone on the waters north of Pointe Aux Barques, Michigan, heading for the protection of Thunder Bay. At 2 am, the ship began its death throes, forcing the crew onto the deck, where many jumped to their deaths in the 34 degree Lake Huron waters. At 2:15 am, the ship broke in two, and the remaining crewmen loaded into a raft on the forward section of the vessel. While they waited for the bow section to sink and the raft to be thrown into the lake, there were shouts that a ship had been spotted off the port bow. Moments later, it was discovered that the looming object was not another ship, but in fact the Morrell's aft section, barreling towards them under the power of the ship's engines. The two sections collided, with the aft section continuing into the distance. In the words of writer Bill Ratigan, the remnants of the vessel disappeared into the darkness "like a great wounded beast with its head shot off".

The SS Edward Y. Townsend had a large crack in its deck from the same storm, It was declared a total loss and was docked for almost two years. It would later be towed to Europe to be scrapped. On its way for scrapping it broke in two and sank in the same vicinity as the R.M.S. Titanic.

The Morrell was not reported missing until 12:15pm the following afternoon, 30 November, after the vessel was overdue at its destination, Taconite Harbor, Minnesota. The U.S. Coast Guard issued a "be on the lookout" alert and dispatched several vessels and aircraft to search for the missing freighter. At around 4:00 pm on 30 November a Coast Guard helicopter located the lone survivor, 26-year-old Watchman Dennis Hale, near frozen and floating in a life raft with the bodies of three of his crewmates. Hale had survived the nearly 40-hour ordeal in frigid temperatures wearing only a pair of boxer shorts, a lifejacket, and a pea coat.

-Wikipedia