Program semantics meets architecture: What if we did not have branches?

Soner Onder
Department of Computer Science
Michigan Technological University
Prologue
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Program semantics meets architecture: \textbf{What if we had branches?}
Is this a serious talk?
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Possibly.
Is this a serious talk?

Possibly.

Are you going to be talking about predication?
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Nihil sub sole novum.
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Why are you giving this talk then?
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To give a new perspective.
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So, let’s start with the question:

Why do we need to use branches in implementing the semantics of our programs?
Turing completeness
Turing completeness

In their seminal paper:

*Flow Diagrams, Turing Machines And Languages With Only Two Formation Rules, Communications of the ACM (CACM), Volume 9 Issue 5, May 1966 (*)&

Corrado Böhm and his student Giuseppe Jacopini proved that *sequencing, selection* and *iteration* are sufficient to simulate any Turing machine.

Corrado Böhm, professor emeritus at the University of Rome "La Sapienza", left us on October 23, 2017 at the age of 94.

He has been an exceptionally talented and creative researcher: his results have deeply influenced the development of theoretical computer science.
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(*) One of two references in Dijkstra’s *Go To Statement Considered Harmful* paper.
Turing completeness

Edsger Wybe Dijkstra:

Guiseppe Jacopini seems to have proved the (logical) superfluousness of the go to statement. The exercise to translate an arbitrary flow diagram more or less mechanically into a jump-less one, however, is not to be recommended. *Then the resulting flow diagram cannot be expected to be more transparent than the original one.* (*)

(*) I highlighted the text

Turing completeness
Question:
Eliminating Go To statements through structured programming does not eliminate branches. We still need to implement structured constructs using branches at the low-level, right?
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Answer:
No, we do not need branches even at the low-level, if we forgo controlling instructions and concentrate only on data values. We only need *gating* and *recursion*.
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Answer:
No, we do not need branches even at the low-level, if we forgo controlling instructions and concentrate only on data values. We only need gating and recursion.

In the rest of the talk, I’ll show that we can efficiently implement:

1. Sequencing in a data-driven manner
2. Selection using gating functions
3. Iteration through recursion

All without branches!

Furthermore, we will end-up with a “more transparent” program than the original program!
Revisiting control-dependence – if-then-else
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It is well-known that branch instructions implement control-dependencies. They can be converted to data-dependencies through if-conversion (predication).

Formally:
   An instruction $j$ is control-dependent on $i$ if the execution of $j$ is controlled by $i$. 
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The code
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\begin{align*}
i & : \text{ If } a < b \text{ then } \\
j & : \quad k = 5 \\
\text{else} & \\
k & : \quad k = 10 \\
\text{= k} & 
\end{align*}
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Selection is implemented in the fetch unit by changing PC.
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If-converted code

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\begin{align*}
  T & : \quad P = a < b \\
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<td>$P : k = 5$</td>
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| $k$: $k = 10$ | $= k$ | $k = \text{cmov}\_
eg P, t$ | $k_2 = \Psi_p (k_0, k_1)$ |
|          | $= k$ | $= k$ | $= k_2$ |

Selection is implemented in the fetch unit by changing PC.
Selection is implemented by controlling writes using predicates.
Selection is implemented by controlling whether cmov writes.
Selection is implemented by a MUX at the ALU inputs.
Revisiting control-dependence – if-then-else

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Selection is implemented by controlling writes using predicates.
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Selection is implemented by a MUX at the ALU inputs.

Both paths are fetched and executed.
Executing both paths

- **Path 1:**
  - $p \implies F \implies x = 5 \implies x = x$
  - $p \implies T \implies x = 10 \implies x = x$

- **Path 2:**
  - $p \implies F \implies x = 5 \implies \ldots \implies x = 10 \implies x = x$

Frequent path:

- $x = 10 \implies \ldots \implies x = x$
Revisiting control-dependence - Gating
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If-converted code

T : P = a < b
P : k = 5
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In eliminating branches through if-conversion, regardless of the type of the conversion, the ability to point to the instruction whose result will be used is lost.

Gating

\begin{align*}
k_0 &= 5 \\
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P &= a < b \\
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As a result, instructions providing the value for each path must be unconditionally executed, in anticipation that its value might be needed.
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- \( k_0 = 5 \)
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As a result, instructions providing the value for each path must be unconditionally executed, in anticipation that its value might be needed.

Corollary:

1. Branch instructions route data through memory by selecting instructions. Only the fetched (i.e., selected) instruction can update memory. The word *selection* in Böhm and Jacopini’s paper must be understood as selecting among data values (even though that is not exactly what they had shown).

2. Predication controls who can update memory, i.e., who should write, but it cannot select instructions.

3. Gating selects among data values, i.e., who should be read, but it cannot select instructions – (is that true?)
Executing in “Reverse”
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\[ k_0 = 5 \]
\[ k_1 = 10 \]
\[ P = a < b \]
\[ k_2 = \Psi_p (k_0, k_1) \]
\[ = k_2 + ... \]

This code is in *single-assignment form*, i.e., every variable is assigned only once.
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Evaluating the value of $k_2$
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This is a functional program and the execution you witnessed is Lazy evaluation!

Only the necessary path has been fetched and executed, and there are no branches, only “function calls”.
Two function calls per instruction

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$k_0()$ { return 5 }
$k_1()$ { return 10 }
P() { return $a() < b()$ }
$k_2()$ { return $\Psi_p(k_0(), k_1())$ }

... { return $k_2() + ...$ }

Evaluating the value of $k_2$
Two function calls per instruction

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... { return $k_2()$ + ... }

Evaluating the value of $k_2$
### Two function calls per instruction

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$k_0()$ { return 5 }

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$P()$ { return $a() < b()$ }

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Evaluating the value of $k_2$
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$k_0() \{ \text{return 5} \}$

$k_1() \{ \text{return 10} \}$

$P() \{ \text{return } a() < b() \}$

$k_2() \{ \text{return } \Psi_T (k_0(), k_1()) \}$

... $\{ \text{return } k_2() + ... \}$

Evaluating the value of $k_2$
Two function calls per instruction

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$k_0()$ { return 5 }

$k_1()$ { return 10 }

$P()$ { return $a() < b()$ }

$k_2()$ { return $\Psi_p (5, k_1())$ }

... { return $k_2() + ...$ }

Evaluating the value of $k_2$
Two function calls per instruction

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Instruction

\[ k_0 = 5 \]
\[ k_1 = 10 \]
\[ P = a < b \]
\[ k_2 = \Psi_P (k_0, k_1) \]
\[ = k_2 + \ldots \]

\[ k_0() \{ \text{return 5} \} \]
\[ k_1() \{ \text{return 10} \} \]
\[ P() \{ \text{return } a() < b() \} \]
\[ k_2() \{ \text{return } \Psi_P() (k_0(), k_1()) \} \]
\[ \ldots \{ \text{return } 5 \} + \ldots \]
Two function calls per instruction

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$k_0() \{ \text{ return 5 } \}$
$k_1() \{ \text{ return 10 } \}$
P() \{ \text{ return } a() < b() \}
$k_2() \{ \text{ return } \Psi_p (k_0(), k_1()) \}$
... \{ \text{ return 5 } \} + ...

A “function call” is nothing but reading a memory location. If the location is full, it returns the data value. If it is empty, it evaluates the instruction, stores the value and makes the location full.
A more complicated example

\[
x_3 = \Psi(p \& r) \mid (\neg p \& q)(x_2, x_1) = x_3
\]
A more complicated example

1. It does not matter how complicated the control-flow structure is. Unlike branches, predicate expressions can be evaluated in parallel.

2. If \((p \land r) \lor (\neg p \land q)\) is true, we’ll fetch and execute \(x_2 = 10\), otherwise we’ll fetch \(x_1 = 5\).
How about loops?
How about loops?

```
sum = 0
i = 0

sum = sum + a[i]
i = i + 1
p = i < 100
p ?

F
T
```
How about loops?

```
sum = 0
i = 0
sum = sum + a[i]
i = i + 1
p = i < 100
p ?
```

- F
- T
How about loops?

\[
\text{sum} = 0 \\
i = 0
\]

\[
\text{sum} = \text{sum} + a[i] \\
i = i + 1 \\
p = i < 100 \\
p \, ?
\]

\[
\text{sum}_0 = 0 \\
i_0 = 0 \\
R_1 = T \\
R_2 = T
\]

\[
i_1 = \Psi_{R_1}(i_0, i_2) \\
\text{sum}_1 = \Psi_{R_2}(\text{sum}_0, \text{sum}_2) \\
\text{sum}_2 = \text{sum}_1 + a[i_1] \\
i_2 = i_1 + 1 \\
p = i_2 < 100 \\
p \, ?
\]

\[
\text{sum}_3 = \eta (\neg p, \text{sum}_2)
\]
How about loops?

\[
\begin{align*}
\text{sum} &= 0 \\
i &= 0 \\
\text{sum} &= \text{sum} + \text{a}[i] \\
i &= i + 1 \\
p &= i < 100 \\
p &? \\
\end{align*}
\]

\[
\begin{align*}
\text{sum}_0 &= 0 \\
i_0 &= 0 \\
R_1 &= \text{T} \\
R_2 &= \text{T} \\
i_1 &= \Psi_{R_1}(i_0, i_2) \\
\text{sum}_1 &= \Psi_{R_2}(\text{sum}_0, \text{sum}_2) \\
\text{sum}_2 &= \text{sum}_1 + \text{a}[i_1] \\
i_2 &= i_1 + 1 \\
p &= i_2 < 100 \\
p &? \\
\text{sum}_3 &= \eta(!p, \text{sum}_2)
\end{align*}
\]

R_1 and R_2 are read-once predicates
How about loops?

The loop in single-assignment form facilitates the conversion to recursion:
1. If we keep branches, the code is forward executable.
2. Alternatively, we can drop the branches and call the exit function of the loop.
3. The exit function “iterates” until the predicate becomes true.
Drop branches and (just for fun) shuffle instructions
Drop branches and (just for fun) shuffle instructions

\[
\begin{align*}
\text{sum}_0 & = 0 \\
\text{i}_0 & = 0 \\
R_1 & = T \\
R_2 & = T \\
\text{i}_1 & = \Psi_{R_1}(\text{i}_0, \text{i}_2) \\
\text{sum}_1 & = \Psi_{R_2}(\text{sum}_0, \text{sum}_2) \\
\text{sum}_2 & = \text{sum}_1 + a[\text{i}_1] \\
\text{i}_2 & = \text{i}_1 + 1 \\
p & = \text{i}_2 < 100 \\
p? & \quad \begin{cases} 
\text{F} & \text{if } p \\text{ holds} \\
\text{T} & \text{if } p \text{ does not hold} 
\end{cases} \\
\text{sum}_3 & = \eta(\neg p, \text{sum}_2)
\end{align*}
\]
Drop branches and (just for fun) shuffle instructions

\[
\begin{align*}
sum_0 &= 0 \\
i_0 &= 0 \\
R_1 &= T \\
R_2 &= T \\
i_1 &= \Psi_{R_1} (i_0, i_2) \\
sum_1 &= \Psi_{R_2} (sum_0, sum_2) \\
sum_2 &= sum_1 + a[i_1] \\
i_2 &= i_1 + 1 \\
p &= i_2 < 100 \\
sum_3 &= \eta (!p, sum_2) \\
\end{align*}
\]

Entry point

\[
\begin{align*}
sum_2 &= sum_1 + a[i_1] \\
i_0 &= 0 \\
R_1 &= T \\
i_1 &= \Psi_{R_1} (i_0, i_2) \\
sum_3 &= \eta (!p, sum_2) \\
p &= i_2 < 100 \\
i_2 &= i_1 + 1 \\
sum_1 &= \Psi_{R_2} (sum_0, sum_2) \\
R_2 &= T \\
sum_0 &= 0
\end{align*}
\]
Drop branches and (just for fun) shuffle instructions

\[
\begin{align*}
\text{sum}_0 &= 0 \\
i_0 &= 0 \\
R_1 &= T \\
R_2 &= T \\
i_1 &= \Psi_{R_1}(i_0, i_2) \\
\text{sum}_1 &= \Psi_{R_2}(\text{sum}_0, \text{sum}_2) \\
\text{sum}_2 &= \text{sum}_1 + a[i_1] \\
i_2 &= i_1 + 1 \\
p &= i_2 < 100 \\
\text{p?} &
\end{align*}
\]

Entry point

\[
\begin{align*}
\text{sum}_2 &= \text{sum}_1 + a[i_1] \\
i_0 &= 0 \\
R_1 &= T \\
i_1 &= \Psi_{R_1}(i_0, i_2) \\
\text{sum}_3 &= \eta (!p, \text{sum}_2) \\
p &= i_2 < 100 \\
i_2 &= i_1 + 1 \\
\text{sum}_1 &= \Psi_{R_2}(\text{sum}_0, \text{sum}_2) \\
R_2 &= T \\
\text{sum}_0 &= 0 \\
\end{align*}
\]

Data-driven sequencing
Drop branches and (just for fun) shuffle instructions

\[
\begin{align*}
\text{sum}_0 &= 0 \\
i_0 &= 0 \\
R_1 &= T \\
R_2 &= T
\end{align*}
\]

\[
\begin{align*}
i_1 &= \Psi_{R_1}(i_0, i_2) \\
\text{sum}_1 &= \Psi_{R_2}(\text{sum}_0, \text{sum}_2) \\
\text{sum}_2 &= \text{sum}_1 + a[i_1] \\
i_2 &= i_1 + 1 \\
p &= i_2 < 100
\end{align*}
\]

\[
\begin{align*}
\text{sum}_3 &= \eta (!p, \text{sum}_2)
\end{align*}
\]

Entry point

\[
\begin{align*}
\text{sum}_2 &= \text{sum}_1 + a[i_1] \\
i_0 &= 0 \\
R_1 &= T \\
i_1 &= \Psi_{R_1}(i_0, i_2) \\
\text{sum}_3 &= \eta (!p, \text{sum}_2) \\
p &= i_2 < 100 \\
i_2 &= i_1 + 1 \\
\text{sum}_1 &= \Psi_{R_2}(\text{sum}_0, \text{sum}_2) \\
R_2 &= T \\
\text{sum}_0 &= 0
\end{align*}
\]

Data-driven sequencing
Drop branches and (just for fun) shuffle instructions

\[
\begin{align*}
\text{sum}_0 &= 0 \\
i_0 &= 0 \\
R_1 &= T \\
R_2 &= T
\end{align*}
\]

\[
\begin{align*}
i_1 &= \Psi_{R_1}(i_0, i_2) \\
\text{sum}_1 &= \Psi_{R_2}(\text{sum}_0, \text{sum}_2) \\
\text{sum}_2 &= \text{sum}_1 + a[i_1] \\
i_2 &= i_1 + 1 \\
p &= i_2 < 100
\end{align*}
\]

Data-driven sequencing
Drop branches and (just for fun) shuffle instructions

\[
\begin{align*}
\text{Entry point:} & \quad \text{sum}_2 = \text{sum}_1 + a[i_1] \\
& \quad i_0 = 0 \\
& \quad R_1 = T \\
& \quad i_1 = \Psi_{R_1}(i_0, i_2) \\
& \quad \text{sum}_3 = \eta(!p, \text{sum}_2) \\
& \quad p = i_2 < 100 \\
& \quad i_2 = i_1 + 1 \\
& \quad \text{sum}_1 = \Psi_{R_2}(\text{sum}_0, \text{sum}_2) \\
& \quad R_2 = T \\
& \quad \text{sum}_0 = 0
\end{align*}
\]

Data-driven sequencing
Drop branches and (just for fun) shuffle instructions

```
sum_0 = 0
i_0 = 0
R_1 = T
R_2 = T

i_1 = \Psi_{R_1}(i_0, i_2)
sum_1 = \Psi_{R_2}(sum_0, sum_2)
sum_2 = sum_1 + a[i_1]
i_2 = i_1 + 1
p = i_2 < 100
p?

F

sum_3 = \eta(!p, sum_2)

T

```

Entry point

```
sum_2 = sum_1 + a[i_1]
i_0 = 0
R_1 = T
i_1 = \Psi_{R_1}(i_0, i_2)
sum_3 = \eta(!p, sum_2)
p = i_2 < 100
i_2 = i_1 + 1
sum_1 = \Psi_{R_2}(sum_0, sum_2)
R_2 = T
sum_0 = 0

p = i_2 < 100

```

Data-driven sequencing
Drop branches and (just for fun) shuffle instructions

Entry point

\[
\begin{align*}
\text{sum}_0 &= 0 \\
i_0 &= 0 \\
R_1 &= T \\
R_2 &= T
\end{align*}
\]

\[
\begin{align*}
i_1 &= \Psi_{R_1}(i_0, i_2) \\
\text{sum}_1 &= \Psi_{R_2}(\text{sum}_0, \text{sum}_2) \\
\text{sum}_2 &= \text{sum}_1 + a[i_1] \\
i_2 &= i_1 + 1 \\
p &= i_2 < 100
\end{align*}
\]

\[
\begin{align*}
\text{sum}_3 &= \eta(!p, \text{sum}_2) \\
p &= i_2 < 100 \\
i_2 &= i_1 + 1 \\
i_1 &= \Psi_{R_1}(i_0, i_2) \\
R_1 &= T
\end{align*}
\]

\[
\begin{align*}
\text{sum}_0 &= 0 \\
\text{sum}_2 &= \text{sum}_1 + a[i_1] \\
i_0 &= 0 \\
R_1 &= T \\
i_1 &= \Psi_{R_1}(i_0, i_2) \\
\text{sum}_1 &= \Psi_{R_2}(\text{sum}_0, \text{sum}_2) \\
R_2 &= T \\
\text{sum}_0 &= 0
\end{align*}
\]

Data-driven sequencing
Drop branches and (just for fun) shuffle instructions

\[
\begin{align*}
\text{sum}_0 &= 0 \\
i_0 &= 0 \\
R_1 &= T \\
R_2 &= T
\end{align*}
\]

\[
\begin{align*}
i_1 &= \Psi_{R_1}(i_0, i_2) \\
\text{sum}_1 &= \Psi_{R_2}(\text{sum}_0, \text{sum}_2) \\
\text{sum}_2 &= \text{sum}_1 + a[i_1] \\
i_2 &= i_1 + 1 \\
p &= i_2 < 100
\end{align*}
\]

The Data-driven sequencing process:

**Entry point**

\[
\begin{align*}
\text{sum}_3 &= \eta(!p, \text{sum}_2) \\
p &= i_2 < 100 \\
i_2 &= i_1 + 1
\end{align*}
\]

**If**

\[
\begin{align*}
\text{sum}_3 &= \eta(!p, \text{sum}_2) \\
p &= i_2 < 100 \\
i_2 &= i_1 + 1 \\
i_1 &= \Psi_{R_1}(i_0, i_2) \\
R_1 &= T \\
R_2 &= T \\
\text{sum}_0 &= 0
\end{align*}
\]

**Else (T)**

\[
\begin{align*}
\text{sum}_3 &= \eta(!p, \text{sum}_2) \\
p &= i_2 < 100 \\
i_2 &= i_1 + 1 \\
i_1 &= \Psi_{R_1}(i_0, i_2) \\
R_1 &= T \\
R_2 &= T \\
\text{sum}_0 &= 0
\end{align*}
\]
Drop branches and (just for fun) shuffle instructions

\[
\begin{align*}
\text{sum}_0 &= 0 \\
i_0 &= 0 \\
R_1 &= T \\
R_2 &= T \\
i_1 &= \Psi_{R_1}(i_0, i_2) \\
\text{sum}_1 &= \Psi_{R_2}(\text{sum}_0, \text{sum}_2) \\
\text{sum}_2 &= \text{sum}_1 + a[i_1] \\
i_2 &= i_1 + 1 \\
p &= i_2 < 100 \\
\text{sum}_3 &= \eta(!p, \text{sum}_2) \\
\end{align*}
\]

Data-driven sequencing

Entry point

\[
\begin{align*}
\text{sum}_2 &= \text{sum}_1 + a[i_1] \\
i_0 &= 0 \\
R_1 &= T \\
i_1 &= \Psi_{R_1}(i_0, i_2) \\
\text{sum}_1 &= \Psi_{R_2}(\text{sum}_0, \text{sum}_2) \\
R_2 &= T \\
\text{sum}_0 &= 0 \\
p &= i_2 < 100 \\
i_2 &= i_1 + 1 \\
\end{align*}
\]

End of first iteration!
Conversion to a “functional program”

\[
\begin{align*}
\sum_0 &= 0 \\
i_0 &= 0 \\
R_1 &= T \\
R_2 &= T \\
i_1 &= \Psi_{R_1}(i_0, i_2) \\
\sum_1 &= \Psi_{R_2} (\sum_0, \sum_2) \\
\sum_2 &= \sum_1 + a[i_1] \\
i_2 &= i_1 + 1 \\
p &= i_2 > 100 \\
\sum_3 &= \eta (!p, \sum_2)
\end{align*}
\]

\[
\text{loop} (i_1, \sum_1) \\
\quad \{ \\
\quad \quad \sum_2 = \sum_1 + a[i_1] \\
\quad \quad i_2 = i_1 + 1 \\
\quad \quad p = i_2 > 100 \\
\quad \quad \text{return } \Psi_p (\sum_2, \text{loop}(i_2, \sum_2)) \\
\quad \} \\
\sum_0 = 0 \\
i_0 = 0 \\
\sum_3 = \text{loop} (i_0, \sum_0)
\]
Conversion to a “functional program”

\[ \begin{align*}
    & \text{sum}_0 = 0 \\
    & i_0 = 0 \\
    & R_1 = T \\
    & R_2 = T \\
    & i_1 = \Psi_{R_1}(i_0, i_2) \\
    & \text{sum}_1 = \Psi_{R_2}(\text{sum}_0, \text{sum}_2) \\
    & \text{sum}_2 = \text{sum}_1 + a[i_1] \\
    & i_2 = i_1 + 1 \\
    & p = i_2 > 100 \\
    & \text{sum}_3 = \eta(\neg p, \text{sum}_2)
\end{align*} \]

\[ \text{loop (}i_1, \text{sum}_1\text{)} \]
\[ \{ \]
\[ \text{sum}_2 = \text{sum}_1 + a[i_1] \]
\[ i_2 = i_1 + 1 \]
\[ p = i_2 > 100 \]
\[ \text{return } \Psi_p(\text{sum}_2, \text{loop}(i_2, \text{sum}_2)) \]
\[ \} \]

\[ \begin{align*}
    & \text{sum}_0 = 0 \\
    & i_0 = 0 \\
    & \text{sum}_3 = \text{loop (}i_0, \text{sum}_0\text{)} \\
    & p? \\
    & \text{F} \\
    & \text{T}
\end{align*} \]

Isn’t this tail-recursion?
The program on the right is a functional program, generated from the imperative program, following a completely mechanical procedure. In this program, sequencing is data-driven, selection is provided by gating functions and iteration is implemented using a special form of tail-recursion, we call “cut-tail” recursion.

Cut-tail is a mirror image of continuation-passing, only in reverse (we forward return points).
“Branch” Prediction

Gating

\[ k_0 = 5 \]
\[ k_1 = 10 \]

P = a < b

\[ k_2 = \Psi_p (k_0, k_1) = k_2 \]

Under Lazy evaluation, there is a pipeline delay between the time the gating function gets the predicate and the appropriate path is fetched. The same is true for the recursive iterator.

The only way to remedy this delay is to predict the predicate. In this case, the function can simultaneously evaluate the predicted path and the predicate expression, provided that evaluation of the predicted path is side-effect free.
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Corollary:
1. It is more appropriate to talk about control-dependence prediction, rather than “branch prediction”.
2. Correlation manifests itself now among predicates. It is more appropriate to talk about correlation in control-dependence as well.
3. **Control-dependence prediction, and exploiting control-dependence correlation are lasting contributions.** They won’t go away whether or not programs are expressed imperatively or functionally (or by using branches or not).
What we have
What we have

We have a “graph solver” which can take any imperative program and generate a functional version of it in the form of a program representation, called Future Gated Single Assignment (FGSA) form:

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We have a complete instruction set in which FGSA programs can be encoded at the machine-level:

What we have
What we have

We have a functioning pipelined processor implementation written in ADL language which gives us a cycle-accurate simulator:

What we have
What we have

Finally, we have a compiler that can compile C programs and generate demand-driven ISA code for inner-most loops.
Open Problems
Open Problems

1. Existing branch predictors rely on existence of *sequential sequencing* (i.e., global branch history). We do not know how to exploit “predicate histories” which are data-driven.

2. It seems that in programs with sufficient ILP, prediction of “forward” branches in this domain may not be necessary, or a non-correlating simple predictor would suffice. On the other hand, we MUST predict the loop back-edges. We do not know how to best do this.
Diversity
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“Besides a mathematical inclination, an exceptionally good mastery of one's native tongue is the most vital asset of a competent programmer.”

— Edsger W. Dijkstra
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A simple Turkish sentence: Eve gideceğim.
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Dissecting:
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Dissecting:

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Ev
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Home
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[Diagram of sentence dissection]
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Ev e gid eceğ
Home to go will
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Home to go will I
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Turkish is primarily a suffix based, “postfix language”. In Turkish, it is natural to say everything in “reverse”!
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