Mathematical Sciences Colloquium
Michigan Technological University
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1:00 P.M.

Fisher Hall 326

Numerical Quadrature over the Surface of a Sphere

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An increasing number of applications, arising for example in geophysics, require PDEs to be solved in spherical geometries. Such calculations often need to be supplemented by numerical quadrature over spherical surfaces, in order to obtain integrated quantities, such as total energy, average temperature, etc. The node sets are typically very large, and feature spatially varying density for improved resolution in critical areas. Until recently, the lack of fast and robust algorithms for generating quadrature weights at scattered nodes over a sphere led to the use of tables with node locations and their associated weights. With increasing problem sizes, this approach becomes impractical:

*Table sizes become excessively large.

*The concept of pre-tabulated nodes is incompatible with problem dependent local node refinement.

Recent advances in computing quadrature weights have included:

*A numerically robust spectrally accurate spherical harmonics-based algorithm costing $O(N^3)$ operations and $O(N^2)$ memory for N quasi-uniform nodes.

*A Radial Basis Function (RBF) kernel-based algebraically $O(1/N^2)$ accurate algorithm, with the cost reduced to $O(N^2)$ operations (but still $O(N^2)$ memory) for N arbitrarily distributed nodes.

We will here describe yet another RBF based approach, which now gives improved $O(1/N^{3.5})$ accuracy at near linear cost: $O(N \log N)$ operations and $O(N)$ memory.